

# Negative Electric Current in Semiconductors

V.I. Yukalov and E.P. Yukalova

*International Centre of Condensed Matter Physics*

*University of Brasilia CP 04513, Brasilia DF 70919-970, Brazil*  
and

*Bogolubov Laboratory of Theoretical Physics*

*Joint Institute for Nuclear Research, Dubna 141980, Russia*

Electric current in semiconductors is considered as a function of time. Nonequilibrium fluctuations of the current are studied for strongly nonuniform samples. It is demonstrated that at special conditions the total electric current through a device can exhibit a negative fluctuation, when the current is directed against the applied voltage.

72.20.-i, 05.60.+w

**keywords:** transport equations, nonuniform semiconductors, electric-current fluctuations

The key to the understanding of the physical properties of semiconductor devices lies in the thorough analysis of equations modelling their behaviour [1]. The equations describing semiconductor transport characteristics are quite complicated, so that usually only stationary regimes can be satisfactory analysed. The situation becomes even more complicated when the distribution of charge carriers is essentially nonuniform. At the same time, strongly nonequilibrium and nonuniform semiconductors can display peculiar features that cannot appear in stationary and uniform samples [2–4].

In this communication we study nonequilibrium transport properties of semiconductor samples with a strongly nonuniform distribution of carrier densities. We show that at some special conditions there can appear a negative electric current, that is, the current directed against the applied voltage. The occurrence of this negative current is a transient effect that can be realized at the initial stage of a nonequilibrium process, while the carrier densities are yet essentially nonuniform. As soon as the charge distribution becomes more uniform, the current returns to normal. Thus, the negative-current effect is displayed as a kind of fluctuation happening because of strong nonuniformity of carriers in a nonequilibrium semiconductor.

The charge carriers are characterized by the densities  $\rho_i = \rho_i(\vec{r}, t)$ , with  $i = 1, 2$ , which are functions of the space variable  $\vec{r}$  and of time  $t$ . Let  $\rho_1 > 0$  and  $\rho_2 < 0$ . Carrier transport is commonly described in terms of the semiclassical drift–diffusion approximation which is the basis of semiconductor device models [1]. This approach rests on the continuity equations

$$\frac{\partial \rho_i}{\partial t} + \vec{\nabla} \cdot \vec{j}_i + \gamma_i \rho_i = 0 \quad (1)$$

and on the Maxwell equation

$$\varepsilon \vec{\nabla} \cdot \vec{E} = 4\pi(\rho_1 + \rho_2), \quad (2)$$

with the electric-current density

$$\vec{j}_i = \mu_i \rho_i \vec{E} - D_i \vec{\nabla} \rho_i, \quad (3)$$

where  $\gamma_i$  is a relaxation width,  $\varepsilon$  is the dielectric permittivity,  $\mu_1 > 0$  and  $\mu_2 < 0$  are the carrier mobilities, and  $D_i$  is a diffusion coefficient.

Adding to the sum of the drift–diffusion currents in eq.(3) the displacement current, one gets the total current density

$$\vec{j}_{tot} = \vec{j}_1 + \vec{j}_2 + \frac{\varepsilon}{4\pi} \frac{\partial \vec{E}}{\partial t}. \quad (4)$$

The total current across a device is given by the integral

$$\vec{J}(t) = \int \vec{j}_{tot}(\vec{r}, t) d\vec{r}, \quad (5)$$

with the integration over the considered sample.

Consider a plane device of the width  $L$  and area  $A$ , being biased with an external voltage  $V_0$ . Then, instead of the vector  $\vec{r}$ , we have one space variable  $x \in [0, L]$ . Define the effective transient time

$$\tau_0 \equiv \frac{L^2}{\mu V_0}, \quad \mu \equiv \frac{1}{2}(\mu_1 + |\mu_2|),$$

where  $\mu$  is the average mobility. It is convenient to scale physical characteristics so that to deal with dimensionless quantities. To this end, we shall measure in what follows the space variable  $x$  in units of  $L$ ; time  $t$ , in units of  $\tau_0$ ; and other physical quantities, in the corresponding units written below:

$$\begin{aligned} \rho_0 &\equiv \frac{Q_0}{AL}, & Q_0 &\equiv \varepsilon A E_0, & E_0 &\equiv \frac{V_0}{L}, \\ \gamma_0 &\equiv \frac{1}{\tau_0}, & D_0 &\equiv \mu V_0, & J_0 &\equiv \frac{Q_0 L}{\tau_0}. \end{aligned}$$

For the plane geometry considered, eq.(1) reduces to

$$\frac{\partial \rho_i}{\partial t} + \mu_i \frac{\partial}{\partial x}(\rho_i E) - D_i \frac{\partial^2 \rho_i}{\partial x^2} + \gamma_i \rho_i = 0 \quad (6)$$

and eq.(2) reads

$$\frac{\partial E}{\partial x} = 4\pi(\rho_1 + \rho_2), \quad (7)$$

where  $0 < x < 1$  and  $t > 0$ .

Assume that at initial time the sample is prepared with a nonuniform distribution of charge carriers

$$\rho_i(x, 0) = f_i(x). \quad (8)$$

There exist several ways of forming samples with nonuniform carrier densities. This can be done, e.g., by means of special procedures of growing a semiconductor with doped layers incorporated during the growth [1]. Another way is to subject the sample to the influence of irradiation by particle or laser beams [3].

Let the considered device be biased with an externally applied voltage  $V_0$ , which, in the dimensionless units accepted, implies the condition

$$\int_0^1 E(x, t) dx = 1. \quad (9)$$

For the electric field we may derive from eq.(7), with condition (9), the functional

$$E(x, t) = 1 + 4\pi \left[ Q(x, t) - \int_0^1 Q(x, t) dx \right] \quad (10)$$

of the charge densities entering through the notation

$$Q(x, t) = \int_0^x [\rho_1(x', t) + \rho_2(x', t)] dx'.$$

The total electric current (5) across the plane device can be written as

$$J(t) = \int_0^1 [j_1(x, t) + j_2(x, t)] dx, \quad (11)$$

where condition (9) is taken into account and

$$j_i = \mu_i \rho_i E - D_i \frac{\partial \rho_i}{\partial x}.$$

From eq. (11) we find

$$J(t) = \int_0^1 [\mu_1 \rho_1(x, t) + \mu_2 \rho_2(x, t)] E(x, t) dx + D_1 [\rho_1(0, t) - \rho_2(1, t)] + D_2 [\rho_2(0, t) - \rho_2(1, t)]. \quad (12)$$

To understand the time behaviour of the electric current, we need to analyse expression (12), in which  $\rho_i$  and  $E$  are defined by eqs.(6) and (7), with the initial condition (8) and the voltage condition (9). Such an analysis can be done resorting to computer calculations. However, before doing this, it is very useful to derive an approximate analytical solution clarifying the physics of processes.

An approximate solution to the system of eqs.(6) and (7) can be obtained employing the method of scale separation [5-7] which is a generalization of the Krylov–Bogolubov averaging method [8] and the of guiding–centre approach [9].

The first step in the method of scale separation [5-7] is the classification of solutions onto fast and slow. In the present case, this is done as follows. Notice that the electric field is expressed as the functional (10) containing the carrier densities integrated over space. Thence,  $E$  should vary in space slower than  $\rho_i$ . On the other hand, the voltage condition (9) shows that the electric field, being averaged over space, does not depend on time. This means that  $E$  can be regarded as a slow function in time. Therefore, the electric field can be classified as a slow solution, with respect to both space and time, as compared to the carrier densities. This allows us to consider eq.(6) for a fast solution  $\rho_i$  treating there  $E$  as a space–time quasi–integral. Solving eq.(6), the found  $\rho_i$  is to be substituted into functional (10), which results in an equation for  $E$  to be solved iteratively.

When solving eq.(6), we continue  $\rho_i$  outside the region  $0 < x < 1$  by defining  $\rho_i$  as zero for  $x < 0$  and  $x > 1$ . As a result we find

$$\rho_i(x, t) = \int_{-\infty}^{+\infty} G_i(x - x', t) f_i(x') dx', \quad (13)$$

where the Green function is

$$G_i(x, t) = \frac{1}{2\sqrt{\pi D_i t}} \exp \left\{ -\frac{(x - \mu_i E t)^2}{4 D_i t} - \gamma_i t \right\}.$$

As an initial condition in eq.(8), it is reasonable to accept the physically realistic case of the Gaussian distribution

$$f_i(x) = \frac{Q_i}{Z_i} \exp \left\{ -\frac{(x - a_i)^2}{2 b_i} \right\} \quad (14)$$

centered at the point  $a_i \in (0, 1)$  and in which

$$Q_i = \int_0^1 f_i(x) dx, \quad Z_i = \int_0^1 \exp \left\{ -\frac{(x - a_i)^2}{2 b_i} \right\} dx.$$

Then, from eq. (13) we obtain

$$\rho_i(x, t) = \frac{Q_i b_i}{Z_i \sqrt{b_i^2 + 2 D_i t}} \exp \left\{ -\frac{(x - \mu_i E t - a_i)^2}{2 b_i^2 + 4 D_i t} - \gamma_i t \right\}. \quad (15)$$

As is evident, at  $t = 0$  solution (15) reduces to the initial condition (14). And at  $t \rightarrow \infty$ , we have

$$\lim_{t \rightarrow \infty} \rho_i(x, t) = 0, \quad \lim_{t \rightarrow \infty} E(x, t) = 1. \quad (16)$$

Now, we have in hands enough information to try to find out whether a negative electric current could really appear, that is, an electric current directed against the applied voltage  $V_0$ . Take, for concreteness, that  $V_0 > 0$ . Then we need to find when  $J(t) < 0$ .

First thing, we immediately may notice, is that if  $\rho_i(x, t)$  is uniform in space then eq. (12) tells us that the electric current is a positively defined quantity  $J(t) = \mu_1 \rho_1 + \mu_2 \rho_2 > 0$ . So, if the electric current has a chance to become negative, this can happen only when  $\rho_i(x, t)$  is nonuniform in space.

From solution (15) it follows that even if the carrier densities are nonuniform at the initial time, nevertheless, they tend to become uniform with time, changing to zero, as  $t \rightarrow \infty$ . Consequently, if a negative electric current could arise,

this might happen only at the initial stage when  $t \ll 1$  and  $\rho_i(x, t)$  are yet essentially nonuniform. Thus, a negative electric current, if any, can occur only as a principally transient effect at time  $t \ll 1$ ; and the space nonuniformity of the initial carrier density  $\rho_i(x, 0) = f_i(x)$  is a necessary condition for this effect.

Any initially nonuniform density, according to form (15), spreads with time because of diffusion and relaxation effects. However, these processes need some time to smooth  $\rho_i$ . Hence, there always can be found such a time  $t \ll 1$  when  $\rho_i$  is yet nonuniform. But diffusion and relaxation shorten the time of a negative-current fluctuation, if such arises. Therefore, the conditions favoring the appearance of the noticeable negative-current effect are  $D_i \ll 1$  and  $\gamma_i \ll 1$ .

After realizing what are the necessary and favoring conditions for the transient effect of a negative-current fluctuation, let us elucidate sufficient conditions for the inequality  $J(t) < 0$ . Since the latter can happen only at  $t \ll 1$  and with a nonuniform density of carriers, let us take  $t = 0$  and the maximally nonuniform initial density

$$\rho_i(x, 0) = f_i(x) = Q_i \delta(x - a_i), \quad (17)$$

resulting from eq.(14) under  $b_i \rightarrow 0$ . Then eq.(12) yields

$$J(0) = \mu_1 Q_1 E(a_1, 0) + \mu_2 Q_2 E(a_2, 0). \quad (18)$$

And for the electric field (10) we get

$$E(x, 0) = 1 + 4\pi Q_1 [a_1 - \Theta(a_1 - x)] + 4\pi Q_2 [a_2 - \Theta(a_2 - x)], \quad (19)$$

where  $\Theta(x)$  is the unit-step function. From eqs.(18) and (19), we find that the inequality  $J(0) < 0$  acquires the form

$$\begin{aligned} & \mu_1 Q_1 \left\{ 1 + 4\pi Q_1 \left( a_1 - \frac{1}{2} \right) + 4\pi Q_2 [a_2 - \Theta(a_2 - a_1)] \right\} + \\ & \mu_2 Q_2 \left\{ 1 + 4\pi Q_2 \left( a_2 - \frac{1}{2} \right) + 4\pi Q_1 [a_1 - \Theta(a_1 - a_2)] \right\} < 0. \end{aligned} \quad (20)$$

Inequality (20) is a sufficient condition for the occurrence of the transient effect of a negative electric current.

There exist a number of cases when inequality (20) holds true. To prove that such cases do exist, consider a particular example when  $a_1 = a_2 \equiv a$ . Then, from eqs.(18) and (19), we derive that  $J(0) < 0$  means that

$$(\mu_1 Q_1 + \mu_2 Q_2) \left[ 1 + 4\pi Q \left( a - \frac{1}{2} \right) \right] < 0, \quad (21)$$

where

$$Q \equiv Q_1 + Q_2. \quad (22)$$

Depending on whether the charge (22) is positive or negative, we have from (21) either

$$a < \frac{1}{2} - \frac{1}{4\pi Q} \quad (Q > 0) \quad (23)$$

or, respectively,

$$a > \frac{1}{2} + \frac{1}{4\pi|Q|} \quad (Q < 0). \quad (24)$$

Remembering that  $0 < a < 1$ , we see that both eqs. (23) and (24) can be satisfied only when

$$|Q| > \frac{1}{2\pi}. \quad (25)$$

Eqs.(23), (24) and (25) are sufficient conditions for the appearance of a negative electric current at the initial stage of the process starting with the nonuniform distribution (17).

To confirm the above analysis, we have solved eqs.(6) and (7) numerically and calculated the electric current (12) as a function of time. We accept the case favoring the appearance of the negative current, when  $D_i \ll 1$  and  $\gamma_i \ll 1$ . The initial condition (8) is taken in the Gaussian form (14). The voltage integral (9) plays the role of a boundary

condition for the electric field. For the carrier densities we may accept the Neumann or Dirichlet boundary conditions [1]. We considered both of them and found that the general behaviour of solutions is practically the same, but the calculational procedure is less stable in the case of Dirichlet conditions. Therefore, for the reason of stability, we preferred the Neumann boundary conditions. The numerical solution of eqs.(6) and (7) was accomplished using the third-order Rusanov finite-difference scheme [10].

The results of numerical calculations are in complete agreement with the necessary and sufficient conditions, obtained above, for the appearance of a negative electric current. Consider, e.g., the case of unipolar or majority-carrier devices such as metal semiconductor field-effect transistors, Schottky varactor diodes, and *GaAs* transferred electron devices [1] that are modelled by a single species set of the basic transport equations. In the case of one type of charge carriers, all processes are absolutely similar with respect to whether this is positive or negative charge. Let us take  $Q_1 \equiv Q \neq 0$  and  $Q_2 = 0$ . Also, for brevity, we shall write  $a_1 \equiv a$  and  $b_1 \equiv b$ . The time behaviour of the electric current (11) is presented in Figs.1 to 5.

Fig.1 demonstrates that if condition (25) does not hold then the negative electric current does not appear. In Fig.2, the charge satisfies condition (25), and the negative current arises provided condition (23) is valid. Fig.3 emphasizes the necessity of an essentially nonuniform distribution of the carrier density at initial time. Fig.4 illustrates that a nonuniform initial density is necessary but not sufficient: to produce a negative current, the location of this nonuniformity is to satisfy condition (23). Fig.5 stresses the importance of the latter condition showing that no negative current appears, even for a large nonuniform initial density, if this condition does not hold true.

Now let us discuss the following two important questions: (i) How the studied electric current  $J(t)$  can be measured? (ii) What could be possible physical applications of the discovered effect?

The quantity that can be measured directly, as is known [1], is the density of the total current across the ends of a device. That is, the directly measurable quantities are the density of current across the left end,

$$J(0, t) \equiv j_{tot}(0, t), \quad (26)$$

and the current density across the right end,

$$J(1, t) \equiv j_{tot}(1, t). \quad (27)$$

How the measurable end currents (26) and (27) are related to the studied total current (11)?

According to the detailed analysis made above, the negative-current effect can arise as a transient phenomenon at the initial stage of the time evolution. Consider the relaxation process in the interval of time  $0 \leq t \leq t_0$ , with  $t_0 \ll \gamma_i^{-1}$ . In the case when  $\gamma_i \ll 1$ , we may have  $t_0 \gg 1$ , so that the considered time interval covers all stages of interest. For this time interval, the evolution equations, following from (1) and (2), in the plane geometry studied are

$$\frac{\partial \rho_i}{\partial t} + \frac{\partial j_i}{\partial x} = 0, \quad \frac{\partial E}{\partial x} = 4\pi(\rho_1 + \rho_2).$$

Basing on these equations, it is straightforward to derive that

$$\frac{\partial}{\partial x} j_{tot}(x, t) = 0, \quad (28)$$

where  $j_{tot}$  is the total current density (4). Eq.(28) tells us that  $j_{tot}(x, t)$ , actually, does not depend on the space coordinate  $x$ . If so, then it is obvious that

$$J(t) \equiv \int_0^1 j_{tot}(x, t) dx = j_{tot}(x, t). \quad (29)$$

Consequently, the end currents (26) and (27) are the same as the total current (29),

$$J(0, t) = J(1, t) = J(t). \quad (30)$$

Thus, the studied total current  $J(t)$  is itself a directly measurable quantity.

In numerical calculations, it is admissible to study any of the currents in eq.(30). However, the integral representation (5) or (29) is more suitable for analytical investigation. This is why we dealt with this representation for deriving the explicit conditions under which the negative-current effect could arise.

It is worth emphasizing that in all our consideration the total current density (4) always was taken in the complete form including the displacement current. The latter nowhere was neglected. The fact that, passing from eq.(5) to

eq.(11), the displacement current formally disappeared in the integrand is due to the advantage of using the integral representation (5) for the total current and because of the voltage condition (9). Really, starting from eq.(5), we have

$$\int_0^1 j_{tot}(x, t)dx = \int_0^1 j(x, t)dx + \frac{\varepsilon}{4\pi} \int_0^1 \frac{\partial}{\partial t} E(x, t)dx,$$

where

$$j(x, t) \equiv j_1(x, t) + j_2(x, t).$$

Invoking the voltage condition (9), we get

$$\int_0^1 \frac{\partial}{\partial t} E(x, t)dx = \frac{\partial}{\partial t} \int_0^1 E(x, t)dx = 0.$$

Consequently, we obtain the identity

$$\int_0^1 j_{tot}(x, t)dx = \int_0^1 j(x, t)dx,$$

Certainly, eqs.(5) and (11) would not be identical if the applied voltage would not be constant. If instead of (9) we would have the voltage condition

$$\int_0^1 E(x, t)dx = \varphi(t),$$

with a time-dependent voltage  $\varphi(t)$  in dimensionless units, then instead of (11) we would get

$$J(t) = \int_0^1 j(x, t)dx + \frac{\varepsilon}{4\pi} \frac{d}{dt} \varphi(t).$$

The fact that under an oscillating applied voltage electric current oscillates as well can surprise nobody. This is why we concentrated on the case of a constant applied voltage. In the latter case, strong fluctuations of electric current are not expected. And the possibility that electric current turns against the applied voltage seems surprising.

The discovered unusual effect of negative electric current is of interest by itself, irrespectively of its feasible applications. Nobody can guarantee in advance what could be the most important use of a new effect. Nevertheless, we may suggest some possible physical applications.

Since the appearance of the negative electric current is closely related to the properties of charge carriers in semiconductor, one can, by measuring this current, extract information about the characteristics of the carriers, for example, about their mobilities.

Another possible application of the effect has to do with the case when the initial charge distribution is formed by irradiating a semiconductor sample by ions. In such a case, the charge distribution is a Gaussian centered at the distance, from the irradiated surface, of a mean free path of ions. The occurrence of the negative electric current essentially depends, according to eqs.(23) and (24), on the location of the initial charge distribution. Therefore, the negative-current effect can be used for measuring the mean free paths of ions in irradiating beams.

One more application is related to the fact that semiconductor devices work very often under the influence of radiation. For instance, such devices can be a part of electronic systems serving near atomic stations or in cosmic space. The influence of radiation can lead to the development of nonuniformities spoiling the work of semiconductor devices. This, in turn, can result in malfunctioning of electronic systems, which can provoke dramatic consequences. Before this happens, it would be desirable to get a warning that nonuniformities due to irradiation have reached the dangerous level. Then one could prevent dramatic accidents. As far as a growing nonuniformity in a device can yield the occurrence of negative electric current, this effect could be used as a kind of warning signaling that the situation is getting dangerous. If inside an electronic system acting under the influence of irradiation one incorporates specially tuned semiconductor devices, based on the effect of negative current, then such devices can play the role of controllers.

It is possible to speculate more suggesting other ways of using the effect. However, we would like to stress it again that a new physical phenomenon is of value as such. All experience teaches us that the most appropriate usage of a newly discovered effect practically never can be predicted in advance.

In conclusion, we have shown that in nonequilibrium semiconductors a negative electric current can arise, being directed against the applied voltage. This effect is principally transient and can appear only at the initial stage of the

process, due to a nonuniform initial carrier density. The necessary and sufficient conditions for the occurrence of this effect are derived. The solution of the transport equations has been done both analytically and numerically.

### Acknowledgement

A grant from the National Science and Technology Development Council of Brazil is appreciated.

---

- [1] C.M. Snowden, Introduction to Semiconductor Device Modelling (World Scientific, Singapore, 1986).
- [2] V.I. Yukalov, in: Ionizing Radiation Effects on Properties of Dielectrics and Semiconductors, ed. M.I. Ryazanov (Atomizdat, Moscow, 1979) p.217.
- [3] A.I. Rudenko and V.I. Yukalov, in: Investigation of Surface and Volume Properties of Solids by Particle Interactions, ed. M.I. Ryazanov (Energoizdat, Moscow, 1981) p. 78.
- [4] V.I. Yukalov, JINR Rapid Commun. 7 (1985) 51.
- [5] V.I. Yukalov, Phys. Rev. Lett. 75 (1995) 3000.
- [6] V.I. Yukalov, Laser Phys. 5 (1995) 970.
- [7] V.I. Yukalov, Phys. Rev. B 53 (1996) 9232.
- [8] N.N. Bogolubov and Y.A. Mitroplosky, Asymptotic Methods in Theory of Nonlinear Oscillations (Gordon and Breach, New York, 1961).
- [9] N.G. Van Kampen, Phys. Rep. 124 (1985) 69.
- [10] V.V. Rusanov, J. Comput. Phys. 5 (1970) 807.

## Figure Captions

### Fig.1

The total electric current as a function of time for the initial carrier density with  $Q = 1/2\pi$  and  $b = 0.05$ , at two different locations:  $a = 0.25$  (curve 1) and  $a = 0.5$  (curve 2). All quantities here and in other figures are given in dimensionless units explained in the text.

### Fig.2

The total electric current as a function of time for  $Q = 0.5$  and  $b = 0.05$ , at different initial locations of carriers:  $a = 0.1$  (curve 1),  $a = 0.25$  (curve 2),  $a = 0.5$  (curve 3) and  $a = 0.75$  (curve 4).

### Fig.3

The time dependence of the total electric current for  $Q = 0.5$  and  $a = 0.25$ , at different widths of the initial nonuniform density of carriers:  $b = 0.05$  (curve 1),  $b = 0.1$  (curve 2) and  $b = 0.5$  (curve 3).

### Fig.4

The total electric current vs. time for  $Q = 3$  and  $b = 0.05$ , at different initial carrier locations:  $a = 0.1$  (curve 1),  $a = 0.25$  (curve 2),  $a = 0.5$  (curve 3) and  $a = 0.75$  (curve 4).

### Fig.5

The total electric current vs. time for  $Q = 3$  and  $a = 0.5$ , at different widths of the initial carrier distribution:  $b = 0.05$  (curve 1) and  $b = 0.5$  (curve 2).